8 – Noise and Measurements

ME-426 – Micro/Nanomechanical Devices

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Introduction

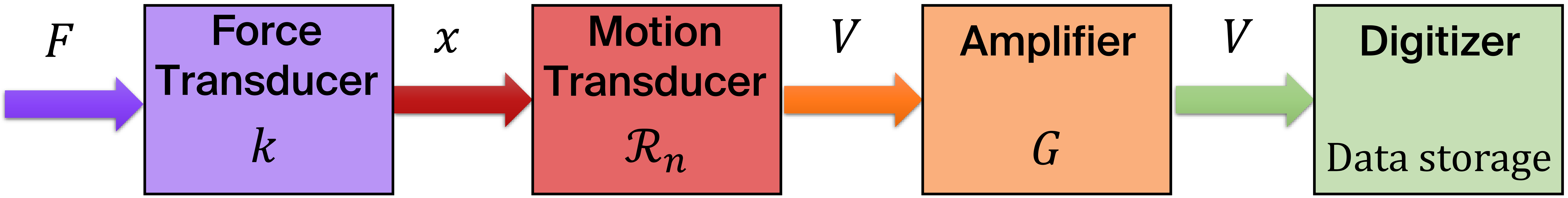
- First order systems
  - Electronic noise
  - Amplifier noise
  - Thermomechanical noise

- Second order systems
  - Frequency noise
  - Allan Deviation
  - Estimation
  - Nonlinearity as a limit

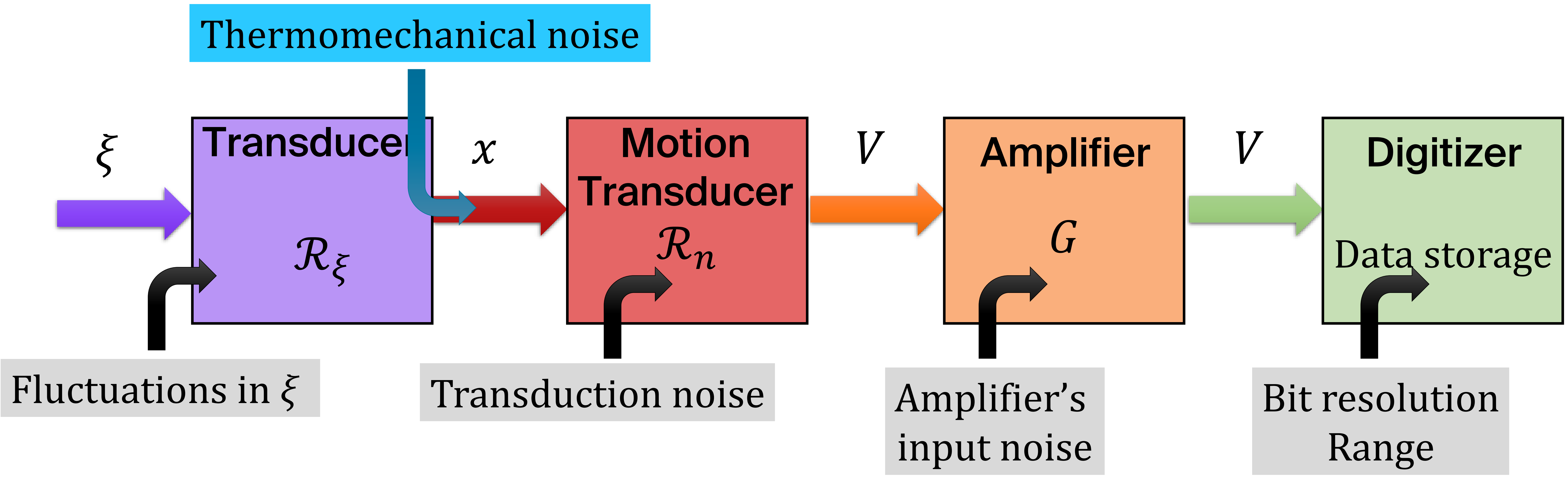
First Order Systems

Direct Measurements

Force detection



General detection – 1st order systems



Noise

- Noise are fluctuations of a given parameter in a random or quasi-random manner
- These fluctuations are added in squares

- We can define the power spectral density:

$$\boldsymbol{\vartheta(t)^2_{noise}} = \langle \vartheta(t)^2 - \langle \vartheta(t) \rangle^2 \rangle = \lim_{T \to \infty} \frac{1}{T} \left[\int_0^T |\vartheta(t)|^2 \, dt - \left| \int_0^T \vartheta(t) \, dt \right|^2 \right] = \int_{-\infty}^{\infty} \boldsymbol{S_\vartheta(f) \, df}$$

- Power spectral density and signal autocorrelation are fourier transforms of one another

- Note that for random noise, autocorrelation is a Dirac delta and PSD is constant for all frequencies
  - Definition of white noise

Electronic noise

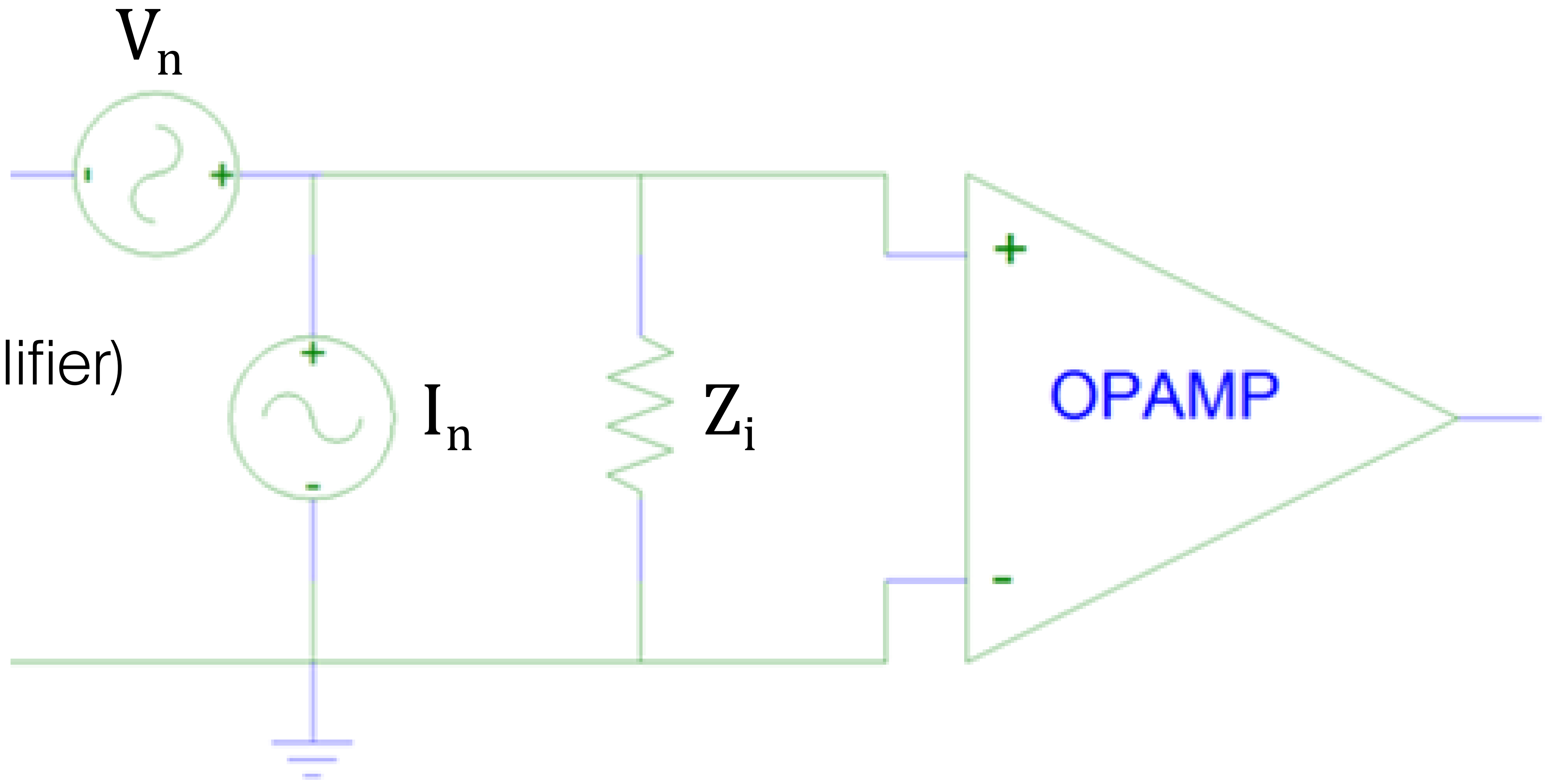
- Johnson-Nyquist noise
  - Thermodynamic fluctuations of electrons and electron states
  - White till a cut-off frequency is reached
  - $\boldsymbol{S_V = 4k_BTR}$
  - For Room Temperature, $R = 50\ \Omega \rightarrow S_V^{1/2} = 0.9\,\frac{\mathrm{nV}}{\sqrt{\mathrm{Hz}}}; S_V = 0.08\,\frac{\mathrm{nV}^2}{\mathrm{Hz}}$

- Shot noise
  - Quantized charge transport
  - $\boldsymbol{S_I = \zeta 2eI}$

- 1/f noise
  - Not clear origin, but clear that is a non-equilibrium noise
  - Can be found in almost every system
  - $\boldsymbol{S_V \propto \frac{1}{f^{\alpha}}}$; with $0.5 < \alpha < 2$ in general. For resistors: $\boldsymbol{S_V \propto \frac{\beta}{n_{carriers}}\frac{V^2}{f}}$

Amplifier noise

- Amplifiers are NOT ideal
- They are composed by transistors, diodes, resistors, etc.
- We *model* their noise by using:
  - Input voltage noise
  - Input current noise
  - Input impedance (noiseless)
  - Noiseless gain stage (ideal amplifier)



Thermomechanical noise

- Fluctuation-Dissipation theorem
  - Every system with dissipation “feels” a fluctuating (random) “force”
  - This theorem is very general but in mechanical devices creates thermomechanical noise

$$m_{eff}\ddot{x}(t) + \frac{m_{eff}\omega_0}{Q}\dot{x}(t) + k_{eff}x(t) = \xi(t)$$

$$\langle \xi(t)\xi(t')\rangle = 2F\delta(t-t'); \qquad \begin{cases} \xi(t) = \displaystyle\int_{-\infty}^{\infty} \xi(\omega)\, \mathrm{e}^{i\omega t}\, \frac{d\omega}{2\pi} \\ \xi(\omega) = \displaystyle\int_{-\infty}^{\infty} \xi(t) e^{i\omega t}\, dt \end{cases}$$

$$\langle \xi(\omega)\xi(\omega')^{*}\rangle = \iint_{-\infty}^{\infty} \langle \xi(t)\xi(t')\rangle \mathrm{e}^{-i\omega t}\, \mathrm{e}^{i\omega' t'}\, dt dt' = 2F\int_{-\infty}^{\infty} \mathrm{e}^{-i(\omega-\omega')t}\, dt = 4\pi F\delta(\omega-\omega')$$

$\xi(t)$ real

Thermomechanical noise

$$m_{eff}\ddot{x}(t) + \frac{m_{eff}\omega_0}{Q}\dot{x}(t) + k_{eff}x(t) = \xi(t)$$

$$X(\omega) = \frac{\frac{\xi(\omega)}{m_{eff}}}{(\omega_0^2 - \omega^2) + \mathrm{i}\frac{\omega_0}{Q}\omega} \rightarrow \langle X(\omega)X(\omega')^*\rangle = \frac{1}{m_{eff}^2}\frac{4\pi F\delta(\omega-\omega')}{(\omega_0^2-\omega^2)^2 + \left(\frac{\omega_0\omega}{Q}\right)^2}$$

$$\langle x^2(t)\rangle = \iint_{-\infty}^{\infty}\langle X(\omega)X(\omega')^*\rangle e^{\mathrm{i}(\omega-\omega')t}\frac{d\omega}{2\pi}\frac{d\omega'}{2\pi} = \frac{F}{\pi m_{eff}^2}\int_{-\infty}^{\infty}\frac{d\omega}{(\omega_0^2-\omega^2)^2 + \left(\frac{\omega_0}{Q}\omega\right)^2} \approx$$

$$\approx \frac{F}{\pi 2\omega_0^2 m_{eff}^2}\int_{-\infty}^{\infty}\frac{d\omega}{(\omega_0-\omega)^2 + \left(\frac{\omega_0}{2Q}\right)^2} = \frac{F}{\pi 2\omega_0^2 m_{eff}^2}\frac{2Q}{\omega_0}\pi = \frac{FQ}{m_{eff}^2\omega_0^3}$$

$Q \gg 1$

Thermomechanical noise

- Using equipartition theorem, we can write that:

$$m_{eff}\ddot{x}(t) + \frac{m_{eff}\omega_0}{Q}\dot{x}(t) + k_{eff}x(t) = \xi(t)$$

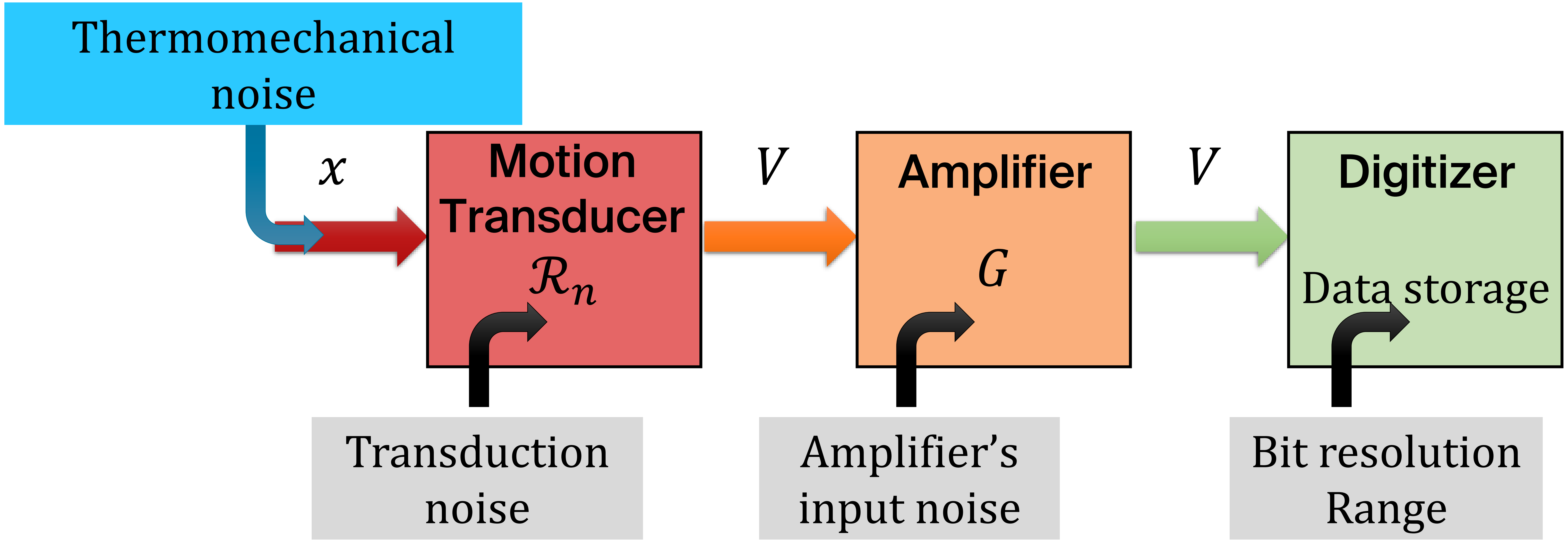
$$\frac{1}{2}k_{eff}\langle x^2(t)\rangle = \frac{k_B T}{2}$$

$$\langle x^2(t)\rangle = \frac{k_B T}{m_{eff}\omega_0^2} = \frac{FQ}{\omega_0^3 m_{eff}^2} \rightarrow F = \frac{m_{eff}\omega_0}{Q}k_B T$$

$$S_x(\omega) = \frac{\frac{k_B T}{Q\omega_0 m_{eff}}}{(\omega_0-\omega)^2 + \left(\frac{\omega_0}{2Q}\right)^2} \rightarrow S_x(\omega_0) = \frac{4Qk_B T}{m_{eff}\omega_0^3}$$

Calibration of the amplitude

- Since the measurements are obtained in Volts, we need a way to calibrate our transduction chain
- For that we use the thermomechanical noise:
  - We first measure the noise in volts - S_V
  - Then we compare with the calculated S_x using the fact that the thermomechanical noise is not white in amplitude



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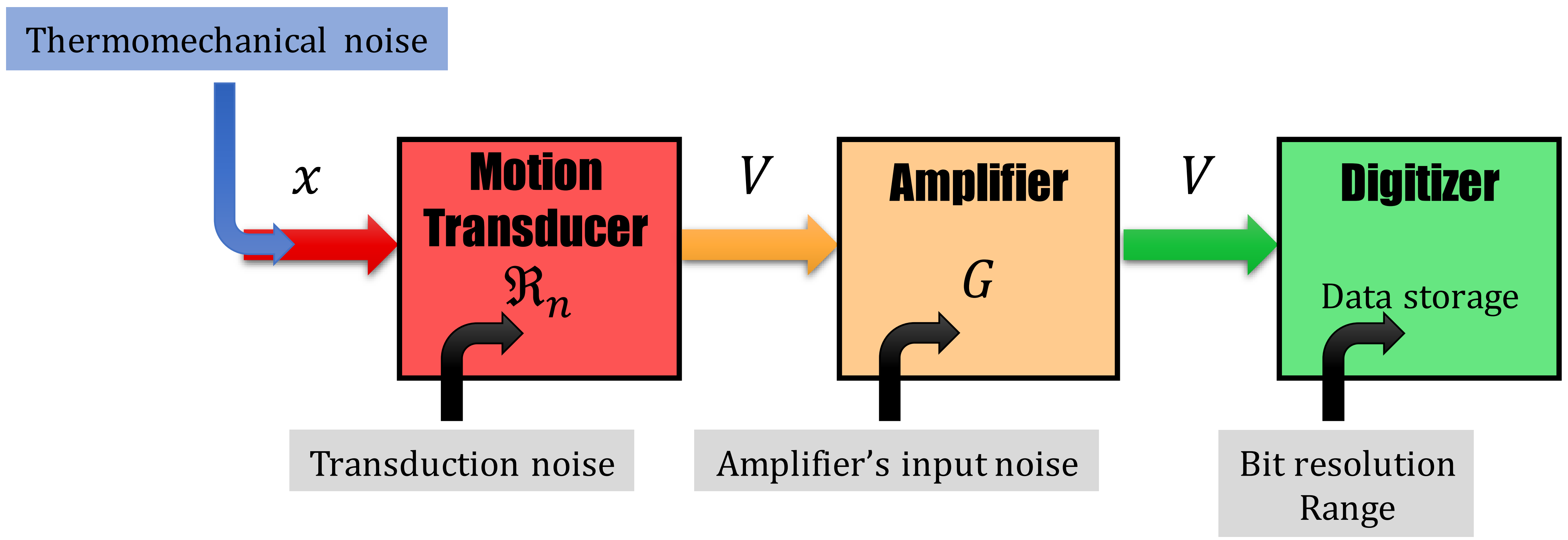
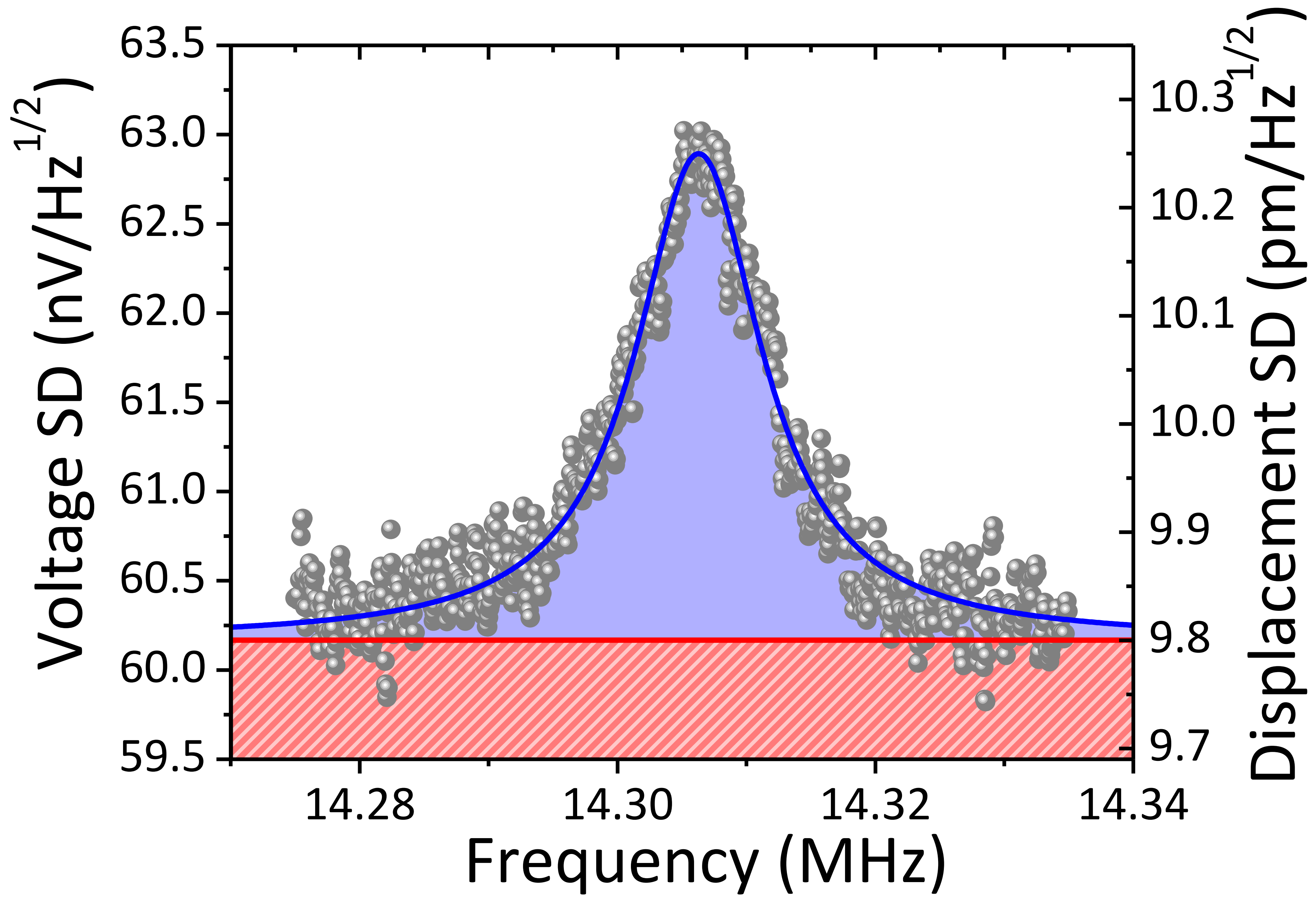
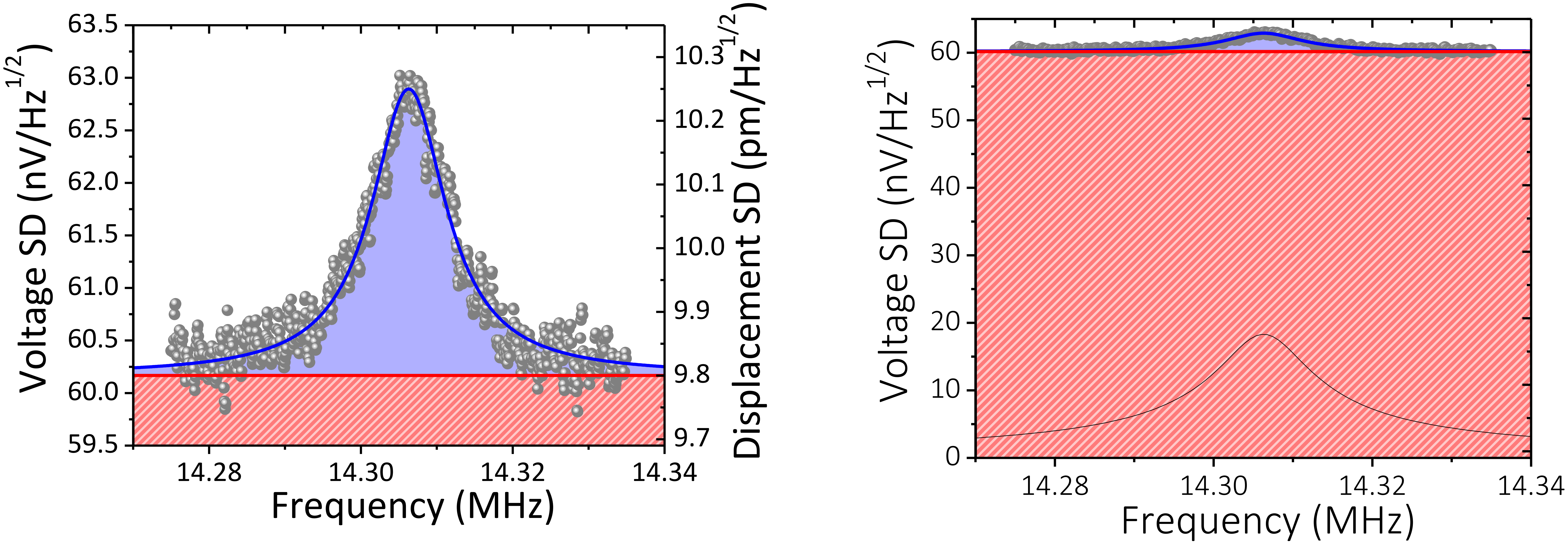
  - $Total\ Gain = R_n G = \sqrt{\frac{S_V}{S_x}}$





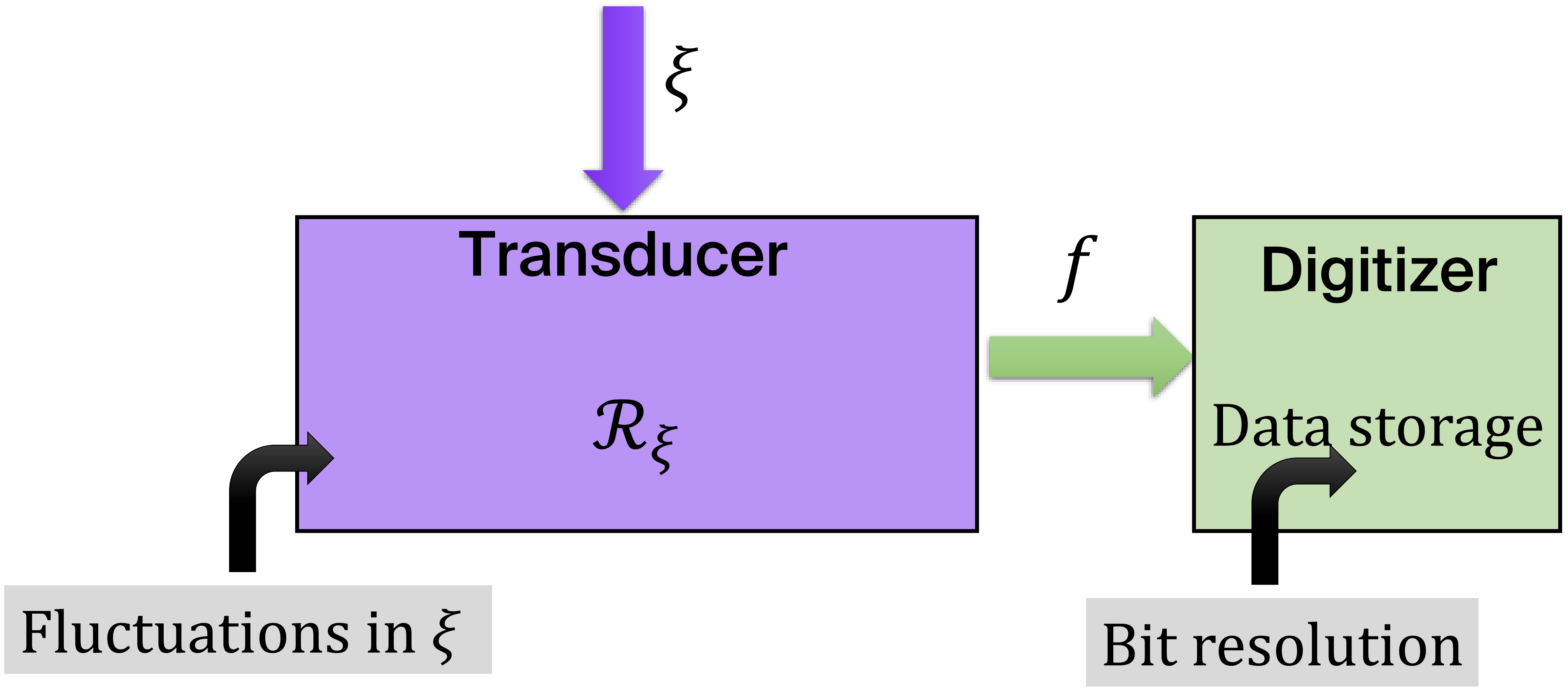
Calibration of the amplitude



Second Order Systems

“Transversal” Measurements

General detection – 2nd order systems



Where are the other noise sources?????

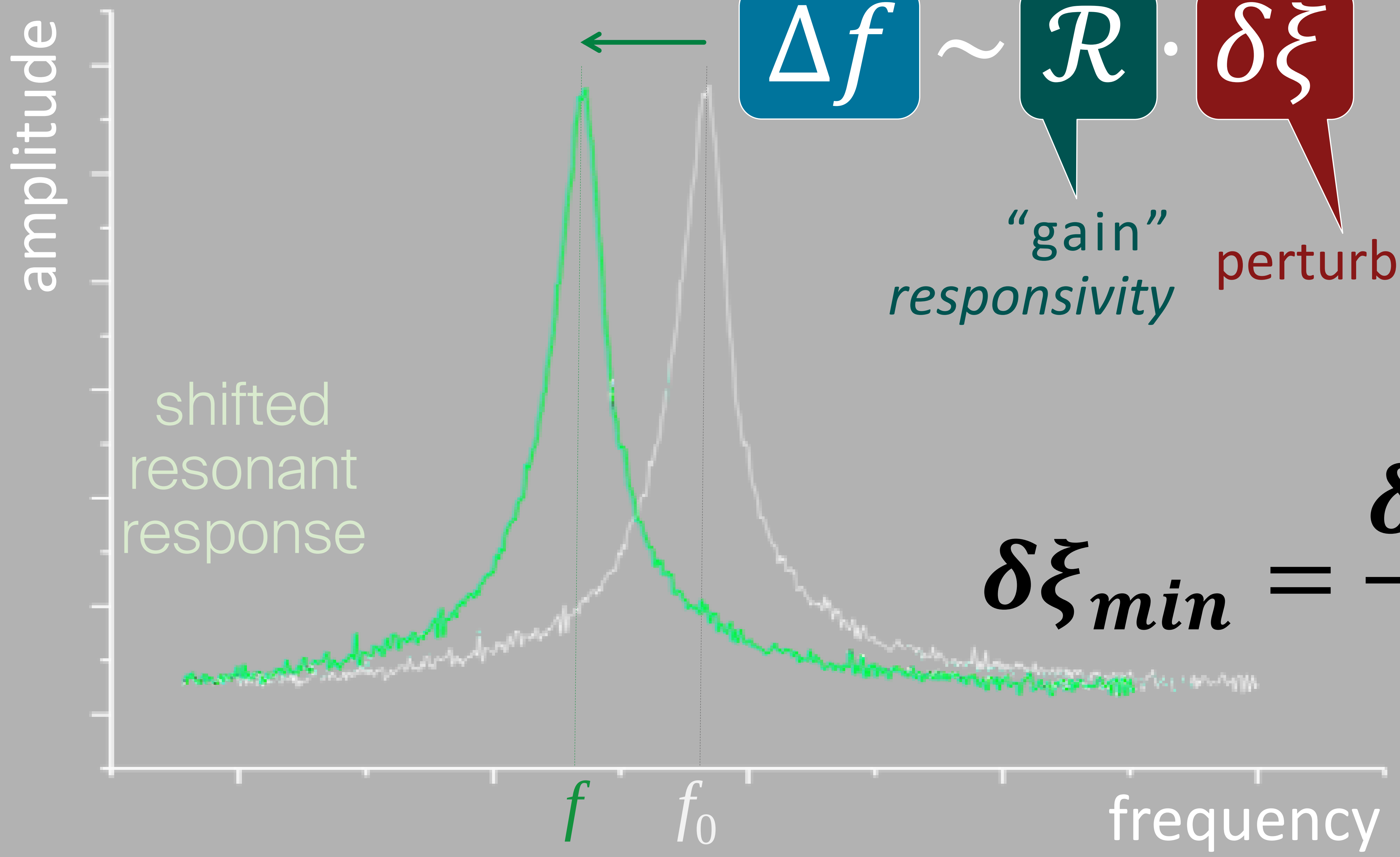
Second order system

frequency shift

$$\Delta f \sim \mathcal{R} \cdot \delta\xi$$

"gain"
responsivity

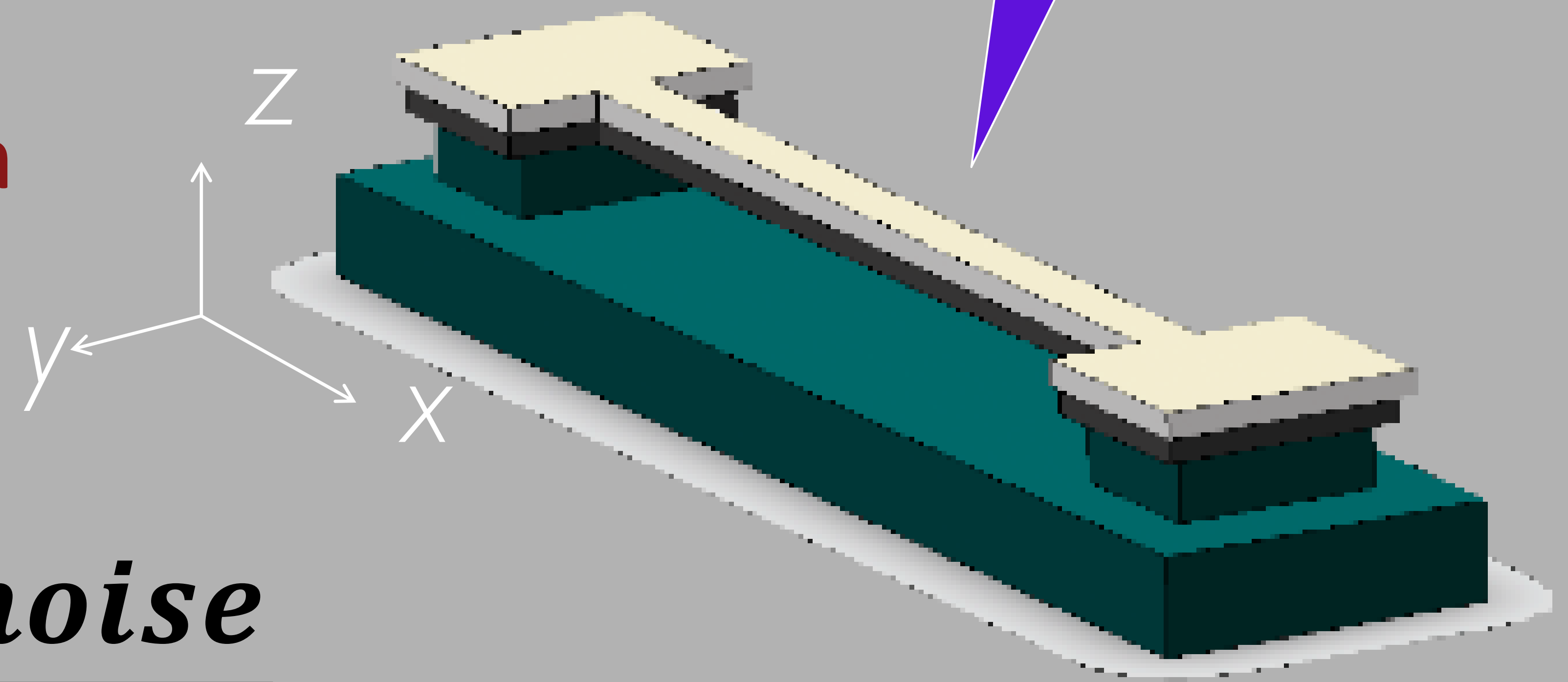
perturbation



$$\delta\xi_{min} = \frac{\delta f_{noise}}{\Re}$$

$\delta\xi$
perturbation
(measurand)

imposed force,
displacement,
added mass,
potential, …

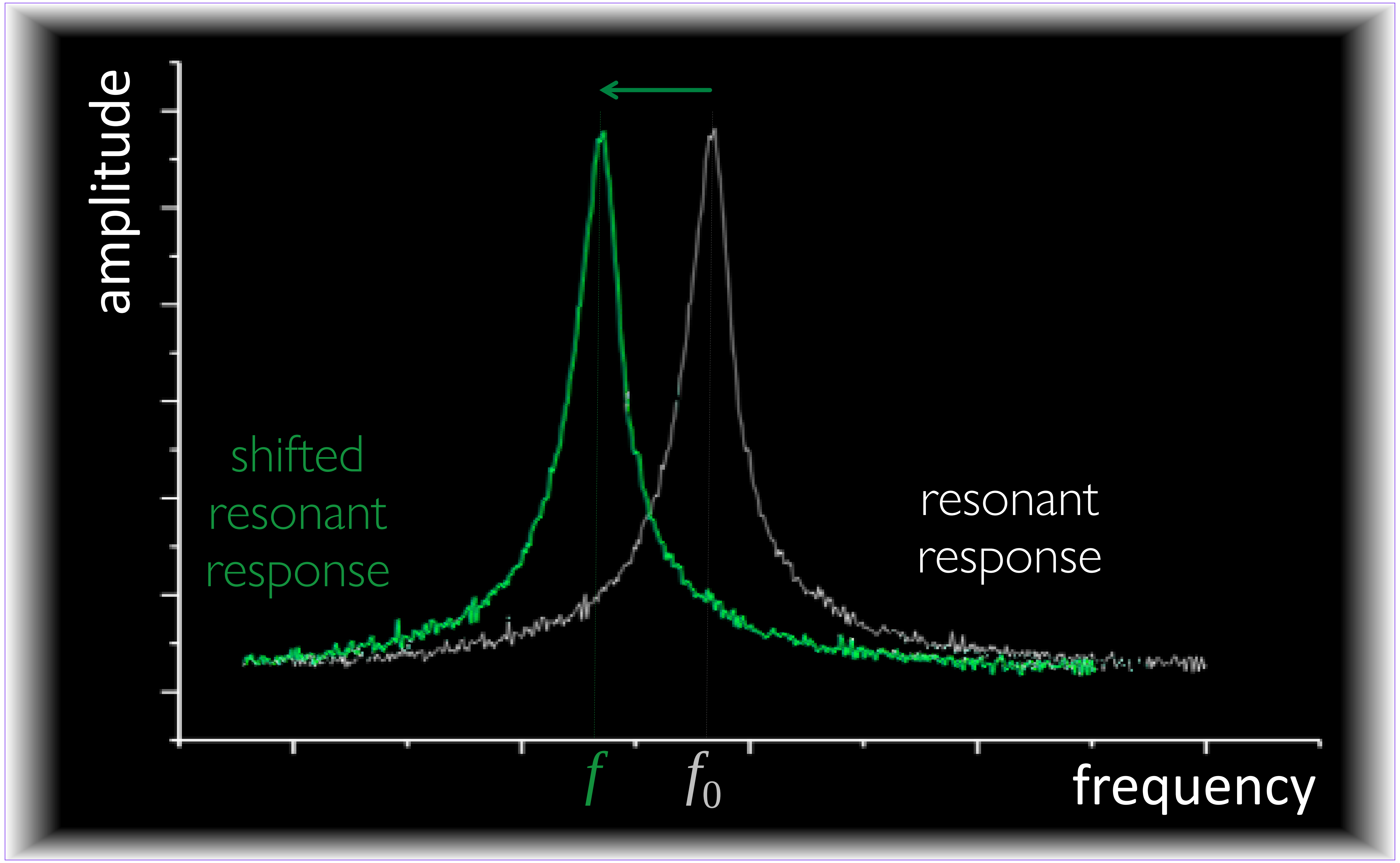


resonant sensor

unperturbed resonance frequency = f_0

Different ways to track the frequency

- Consecutive frequency sweeps to unveil resonance peaks
  - Slow
  - Fitting reduces noise



Different ways to track the frequency

- Consecutive frequency sweeps to unveil resonance peaks
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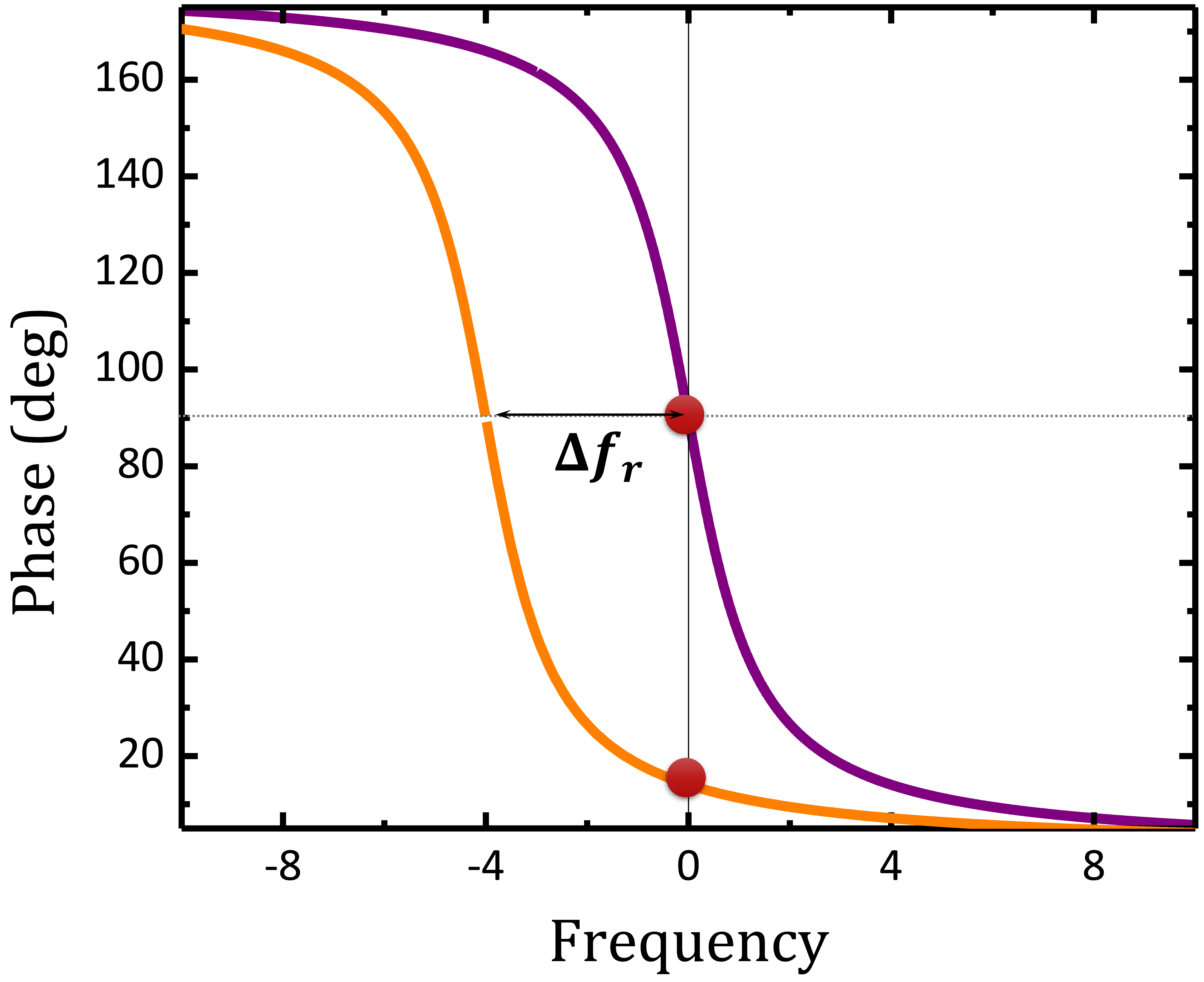
- PLL
  - Fast
  - Easier to calculate limit
  - Noisier?



- Introduction
- How to measure?

Different ways to track the frequency

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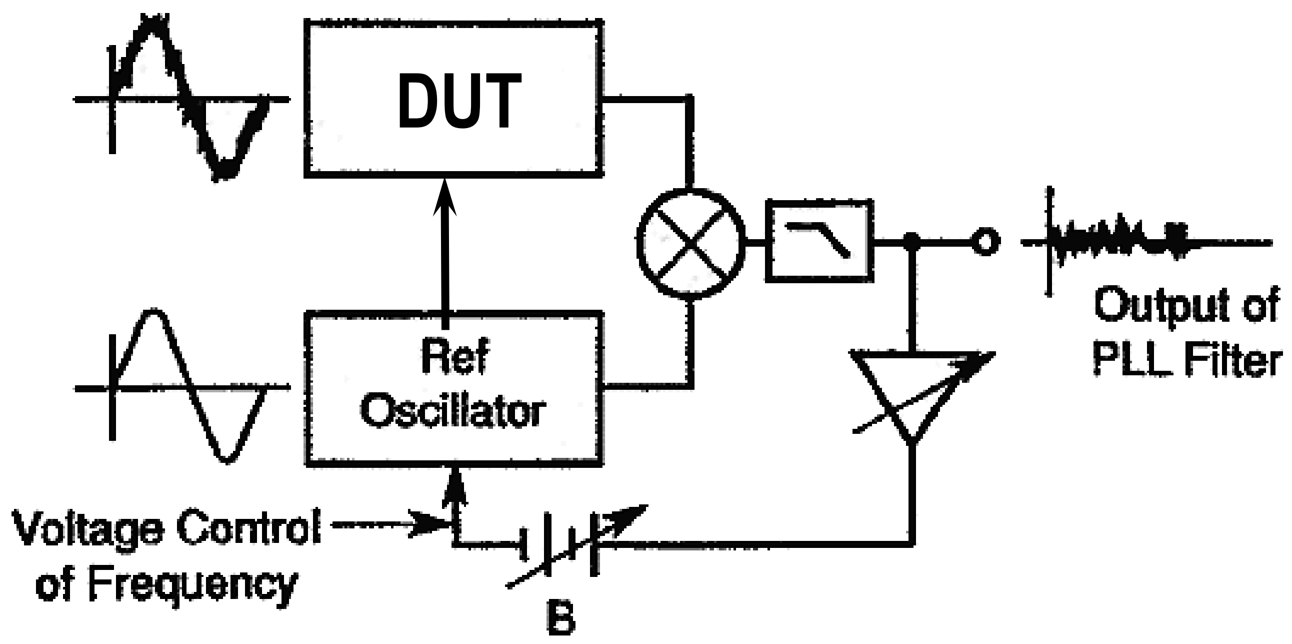
Loose PLL



Tight PLL : Monitor the frequency of the Reference Oscillator

Different ways to track the frequency

- Consecutive frequency sweeps to unveil resonance peaks
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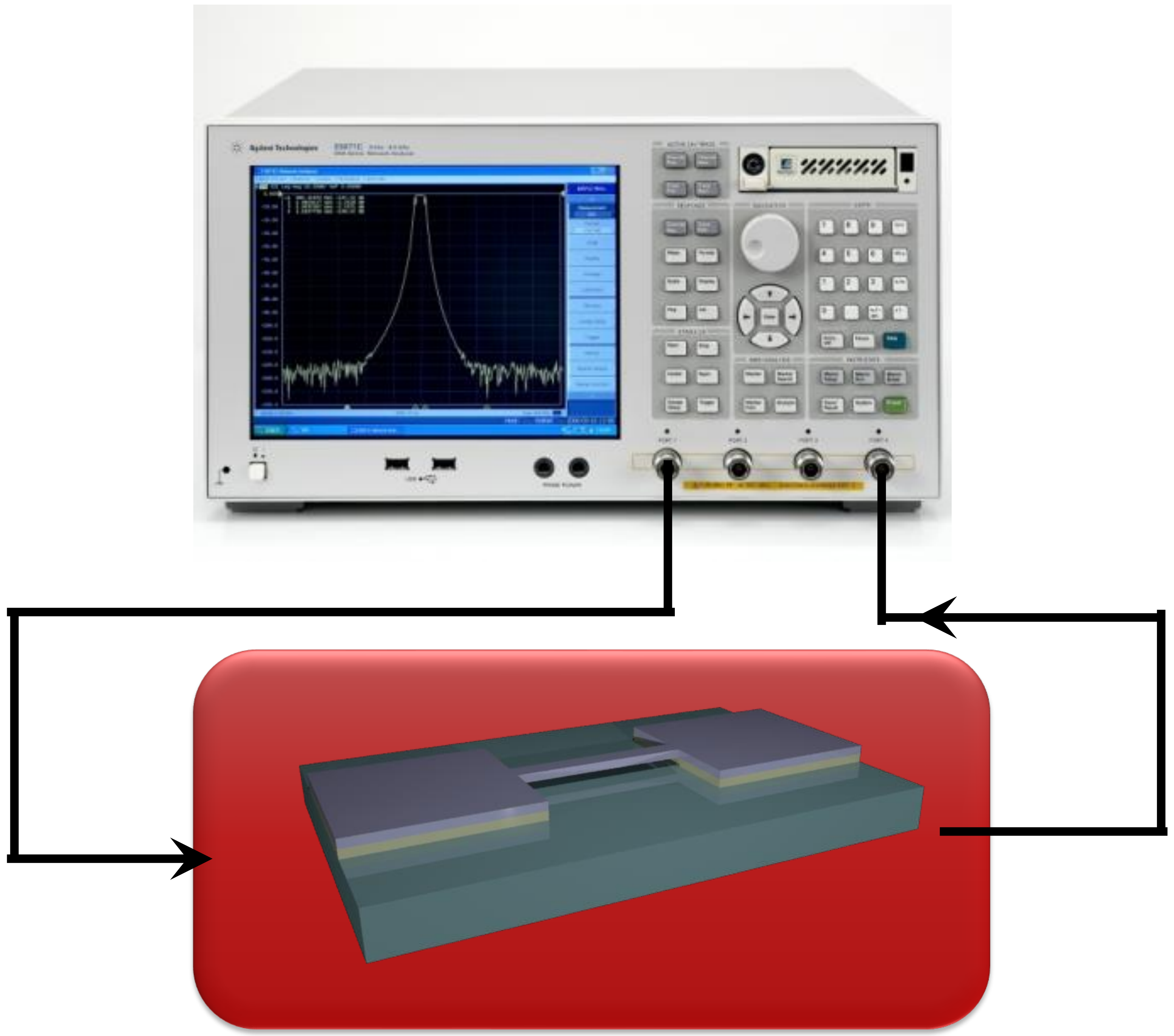
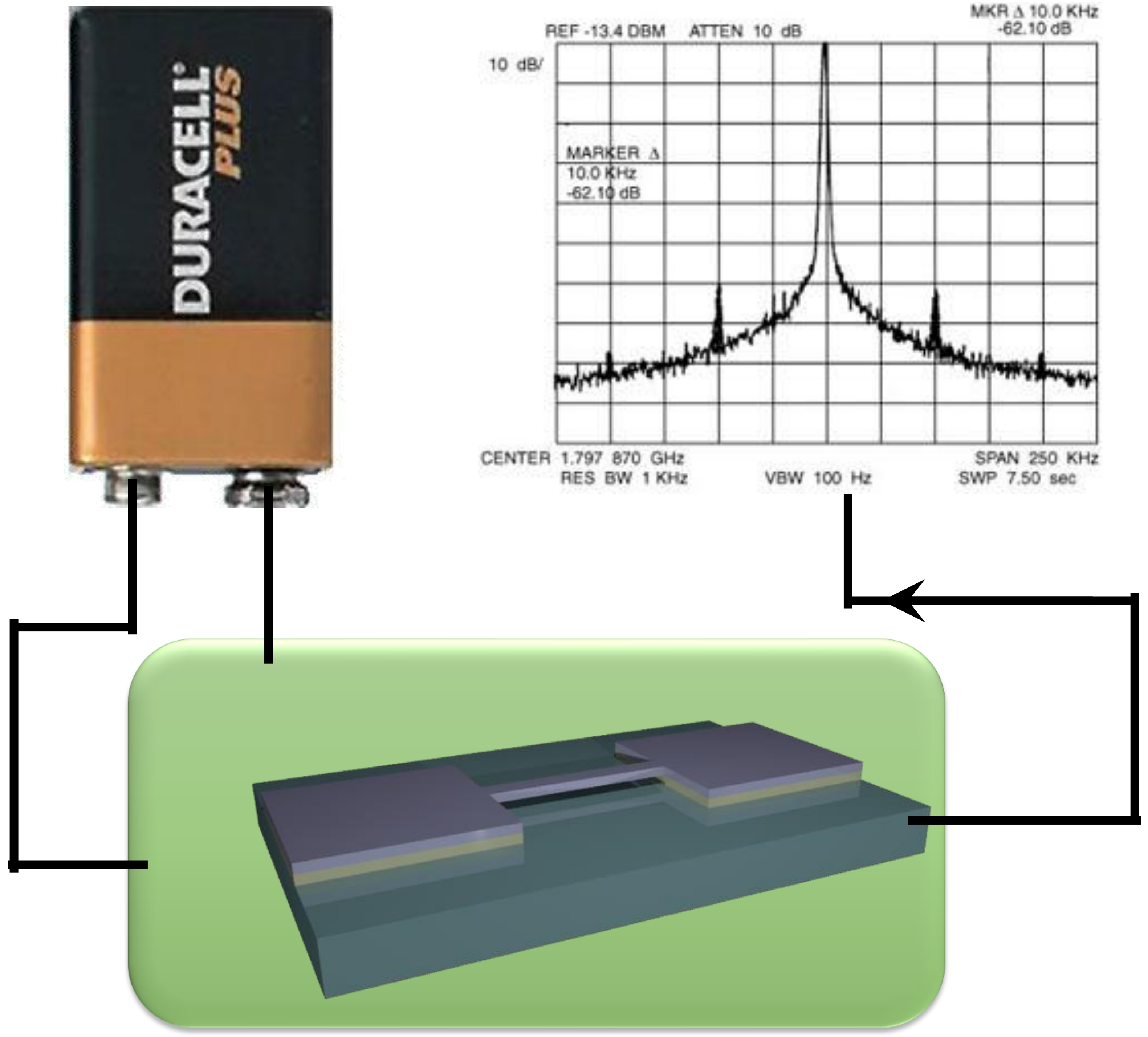
- Oscillator (self sustained)
  - Fastest
  - Noisiest due to phase freedom?

Resonators & oscillators

- Resonators: passive, need of a external AC source
- Oscillators: active, only DC source, AC output





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Limit of detection

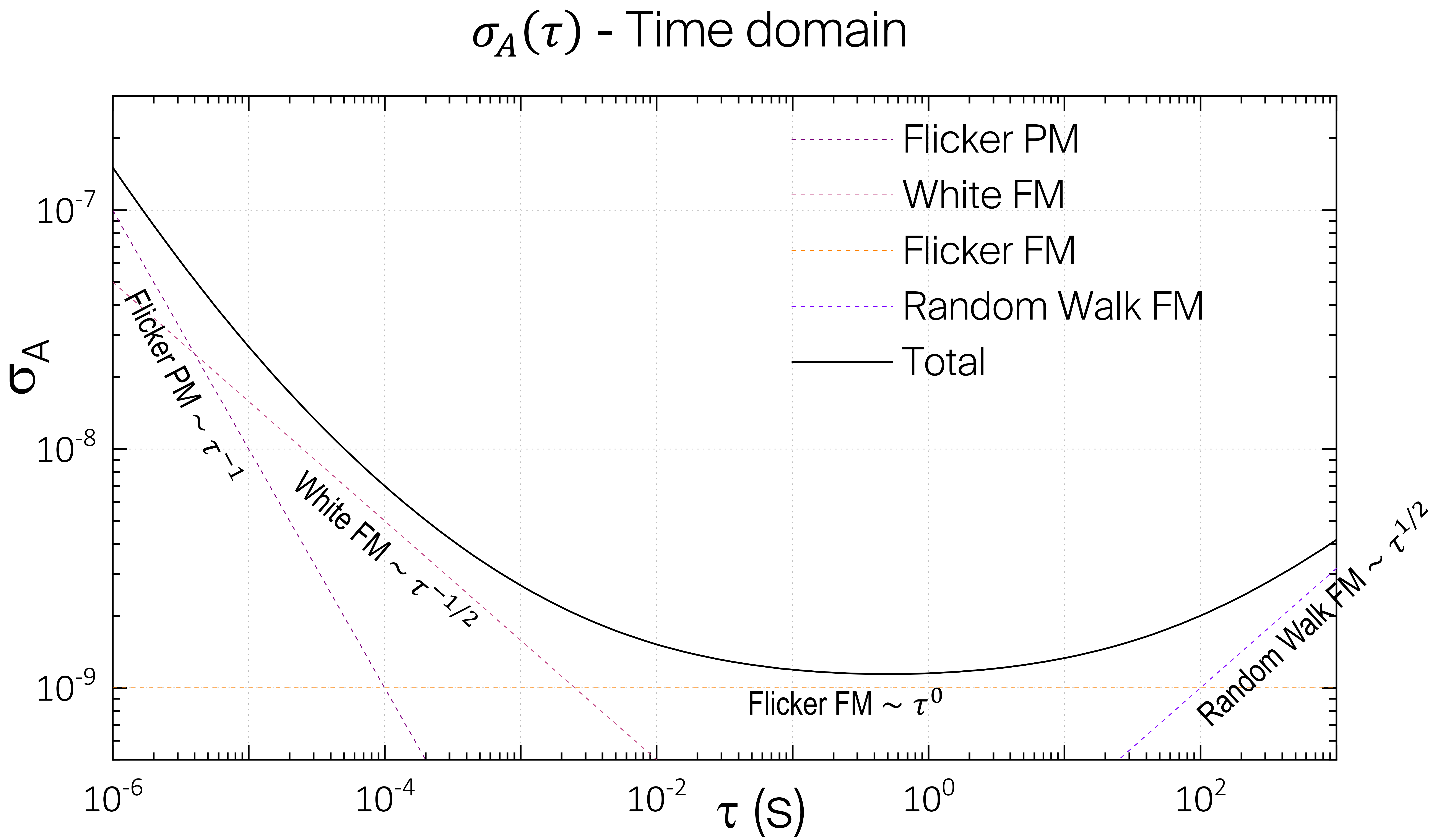
$$\mathcal{R}_\xi = \frac{\partial f}{\partial \xi} \text{ – } \textbf{Responsivity}$$

$$\delta\xi_{min} = \frac{\partial \xi}{\partial f}\delta f_{min} \qquad \rightarrow \delta\xi_{min}(\tau) = \frac{f_0}{\Re_\xi}\sigma_A(\tau)$$

$$\sigma_A^2(\tau) = \frac{1}{2}\left\langle \frac{\left(\bar{f}_{k+1,\tau} - \bar{f}_{k,\tau}\right)^2}{f_0^2} \right\rangle; \qquad \bar{f}_{k,\tau} = \frac{1}{\tau}\int_{t_k}^{t_k+\tau} f(t)\, dt$$

D. Allan, et al., Standard terminology for fundamental frequency and time metrology, 42nd FCS, 1988

Limit of detection



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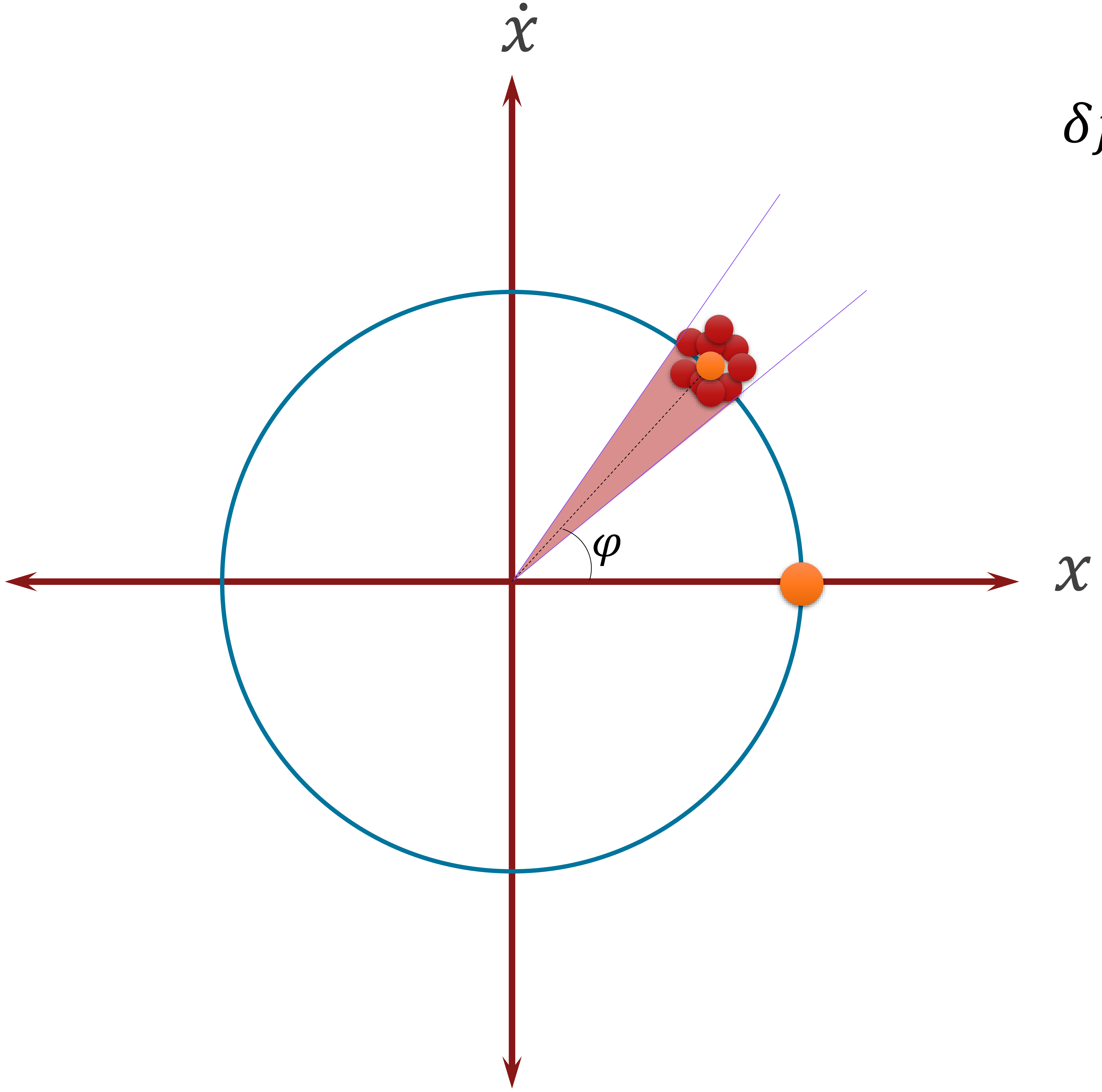
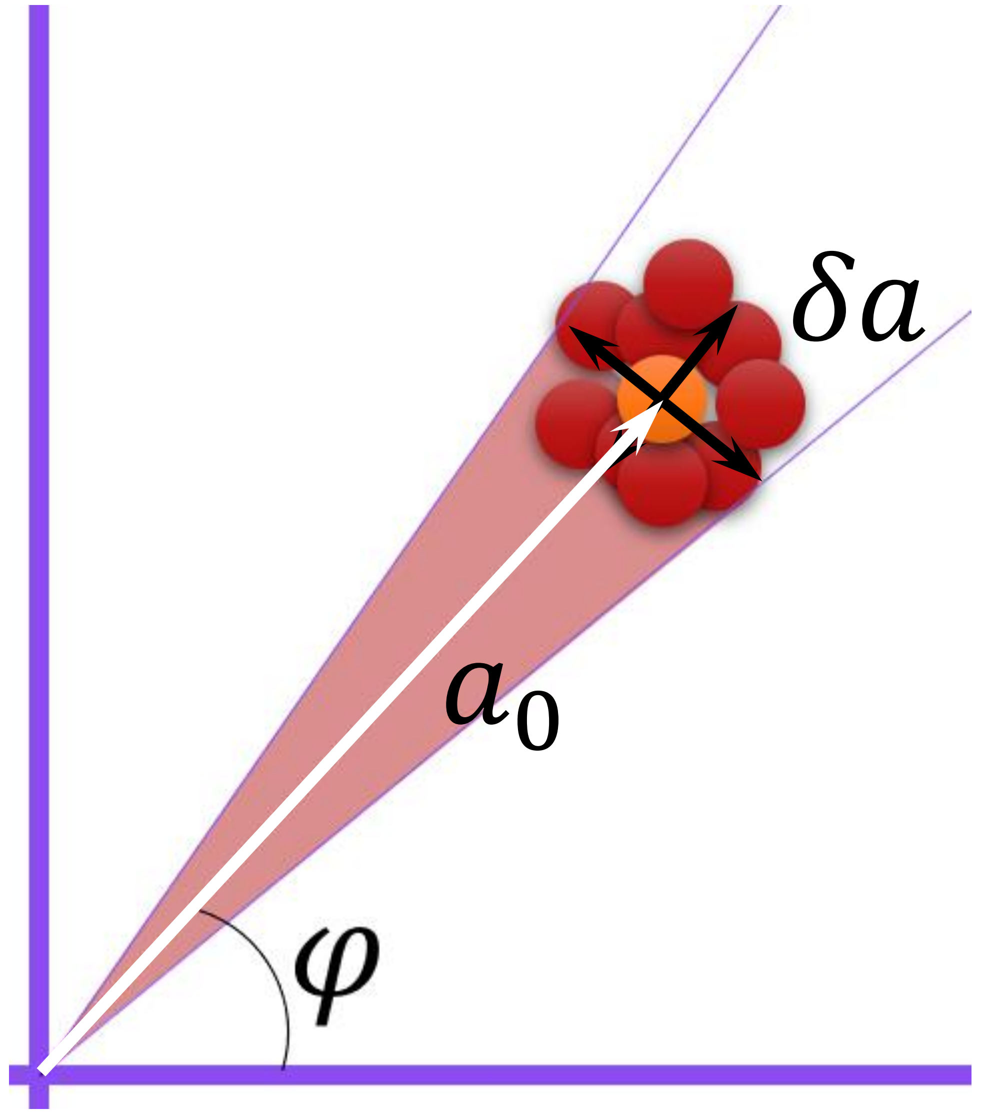
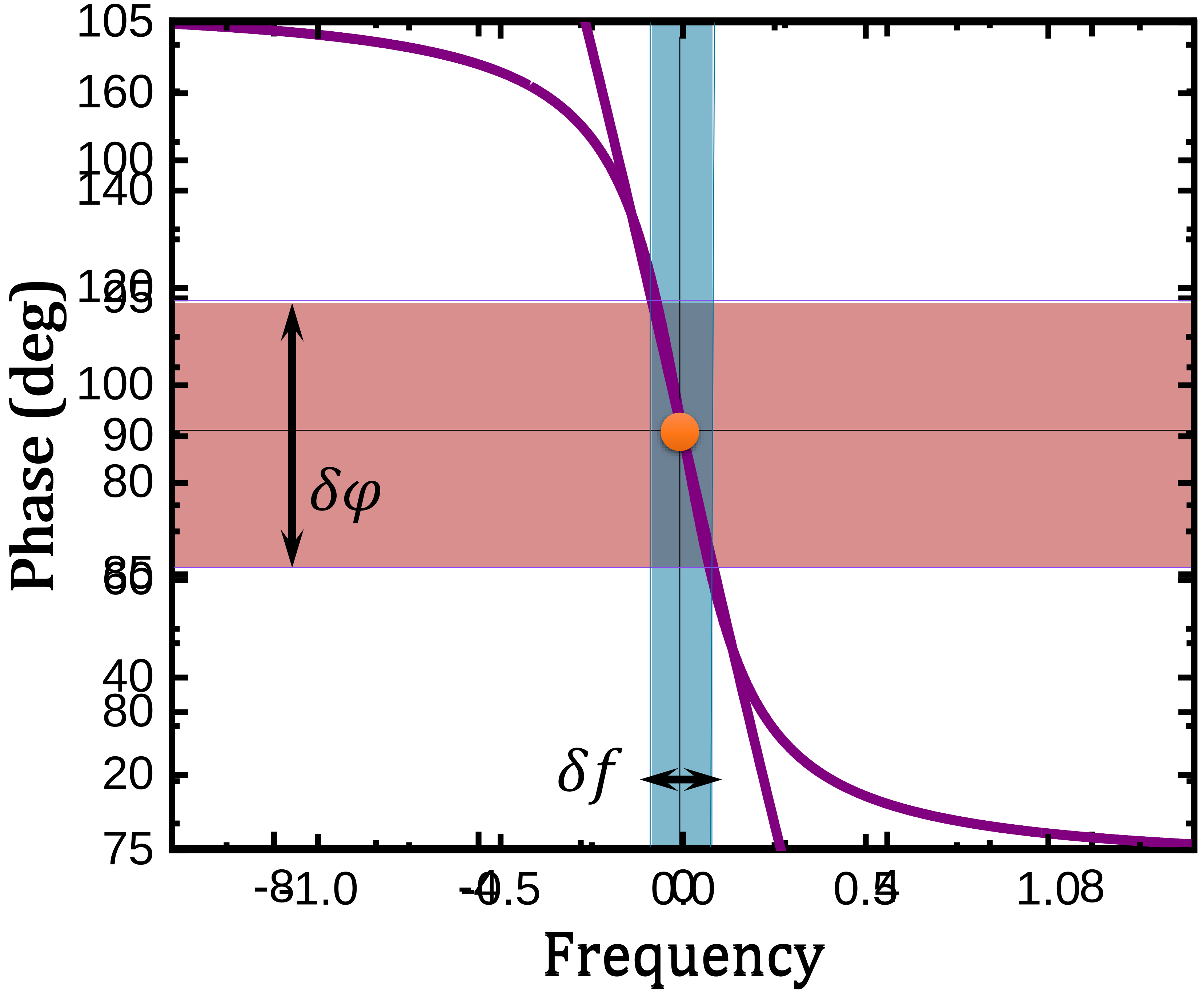
Noise estimation (White noise)



$$\delta f \sim \frac{f_0}{Q}\,\delta\varphi$$





How do we calculate the limit of detection?

- To calculate either $\sigma_A(\tau)$ or $S_\phi(f)$ we need to know the noise source
- In general it is difficult to identify, that is why we only have a formula for the cases of White Frequency Modulation (the same as Phase Modulation Random Walk) and Flicker FM with origin in a phase modulation fed-back.

$$\sigma_A(\tau) = \frac{1}{Q}\left(\frac{E_{Noise}}{E_{Osc}}\right)^{1/2}$$

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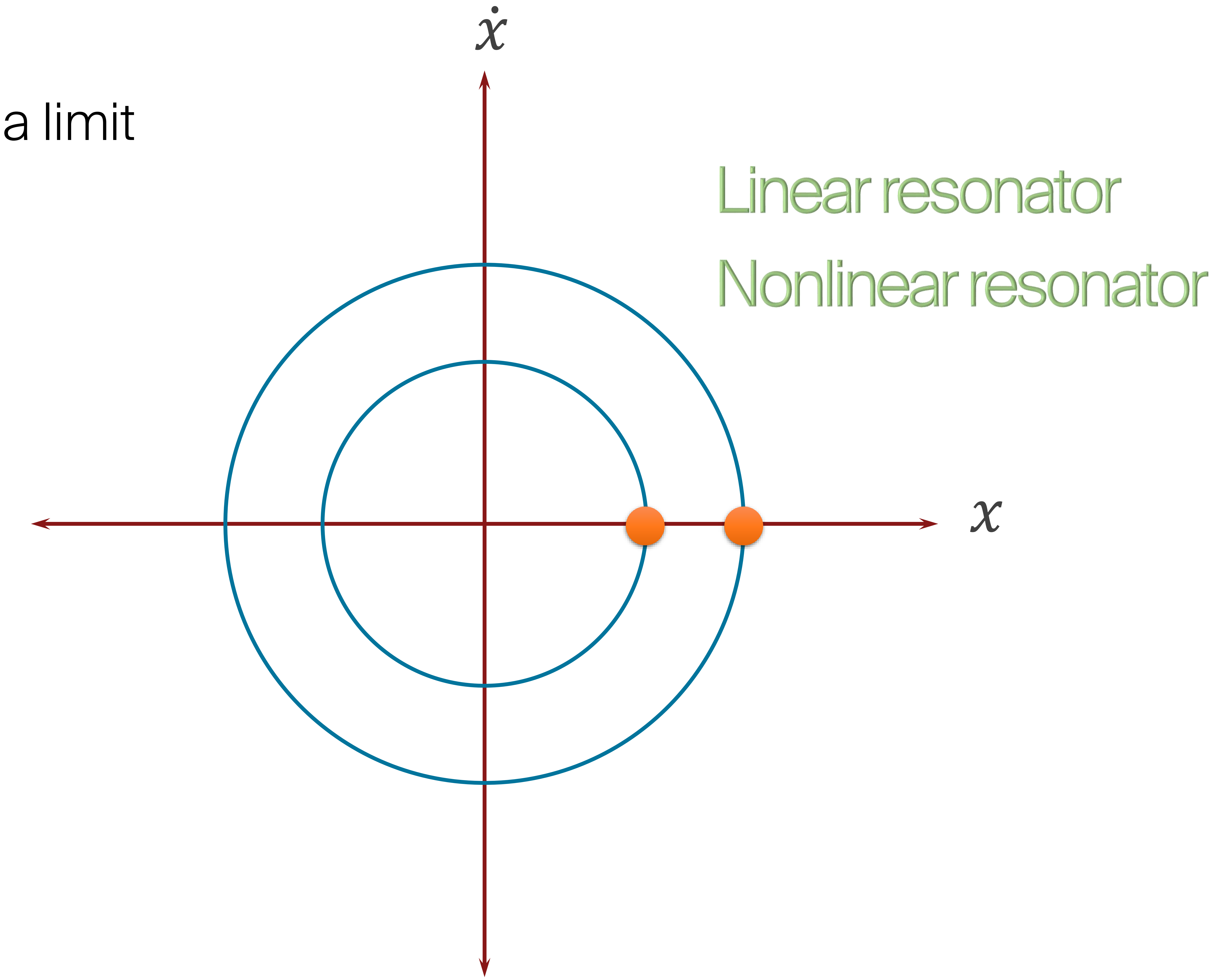
Why can't we bring the Limit of Detection to zero then?

Why not going NL? – A graphical view

- Nonlinearity as a limit



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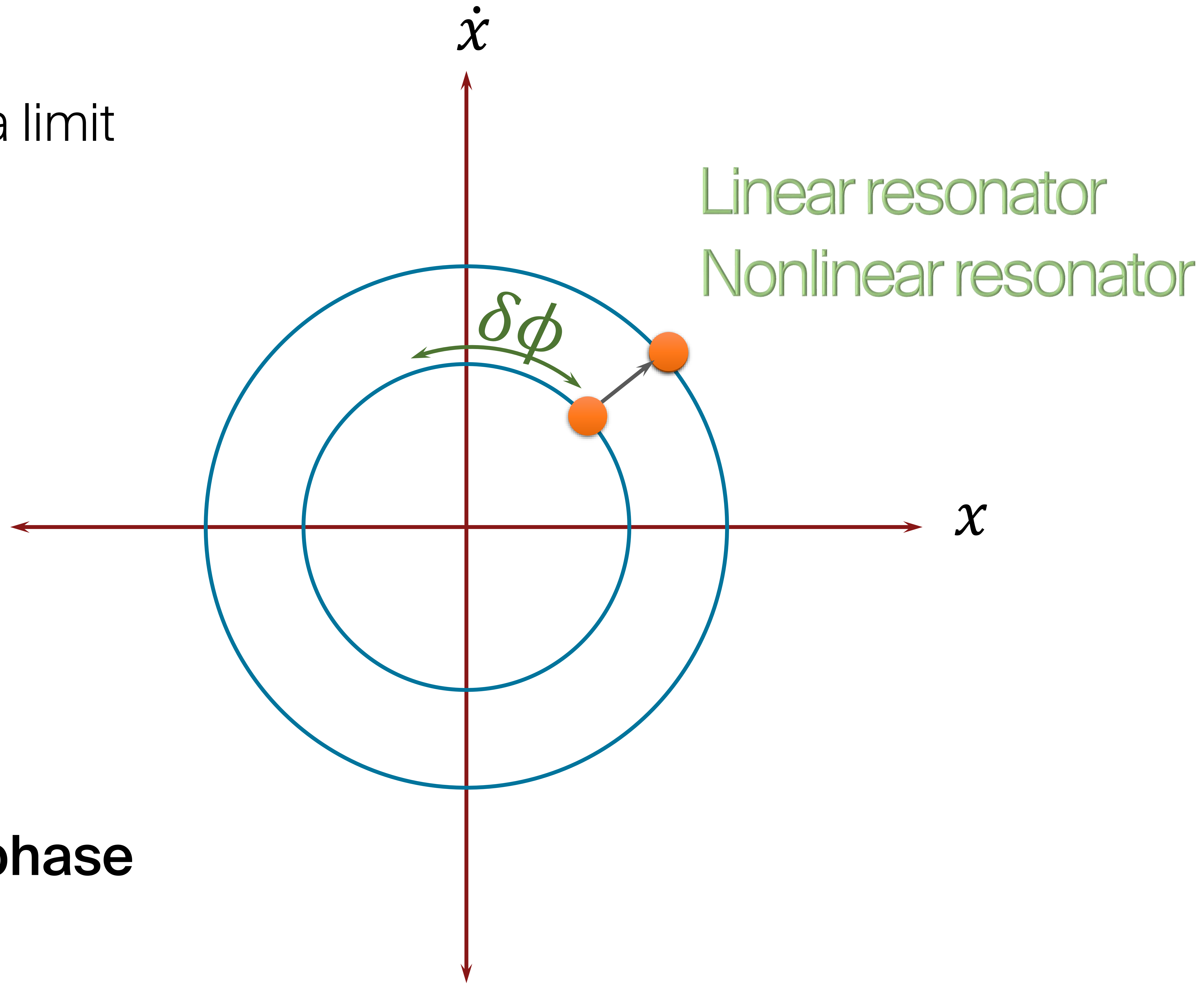

Amplitude-phase conversion